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Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.

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This is a time of transition for Education Notes as Jennifer Hyndman completes her eighth and final year as Co-Editor. A note of appreciation follows the opening article in this issue of Education Notes. Contributions from people in various facets of the mathematical community are welcomed. An example is featured here on the theme of computational thinking.

Computational Thinking: In our Undergraduate Mathematics Programs?

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'Computational Thinking' is becoming increasingly included in schools around the world. For example, England, France, New Zealand, and Australia have integrated it in their mandatory school education, either as a separate field of its own, as a part of mathematics courses, or transversal to curricula. Some of our provinces (e.g., British Columbia and Nova Scotia) are also following this recent trend. In fact, when reflecting on the progress of reform efforts between 2006 and 2016, computer scientist Jeanette Wing predicted that "Computational thinking will be a fundamental skill used by everyone in the world by the middle of the 21st century. By fundamental, I mean as fundamental as reading, writing, and arithmetic" (2016, p. 10). She goes on to mention the international phenomena of greater enrolment in computer science departments, and more availability of computer science courses for non-majors. But what is happening in our undergraduate mathematics programs across Canada? Should mathematics departments also adapt to this change? If so, how?

To start with, what is 'computational thinking'? Wing (2014) defines it as the thought processes involved in formulating problems and expressing their solutions in ways that enable computers (humans or machines) to effectively carry out the solving process. In other words, computational thinking is "thinking *like* a computer scientist". This does not deny the existence of an overlap with other kinds of thinking: for instance, with mathematicians' general problem solving processes, engineers' methods of designing complex systems under real-world constraints, and scientists' approaches to understanding computability, intelligence, and behaviour (Wing, 2008). It also, unavoidably, calls for a reflection on "computer programming", not as a suitable synonym, but as a central activity to supporting the cognitive tasks involved in computational thinking (Grover & Pea, 2013, p. 40).

Suppose we leave the "should we" question for now, and begin by looking at the "how could we". Then it seems natural to start by turning our gaze upon ourselves: How do we, 'research mathematicians', integrate computational thinking in our research work? Laura Broley's (2015) master's thesis gives us some indication. Her interviews with fourteen Canadian mathematicians allowed her to document and analyze examples of how pure and applied mathematicians create and use computer programs to calculate, simulate, visualize, or experiment, whether to model phenomena, develop conjectures, or build tools for solving large classes of problems. Moreover, in 2016, mathematicians leading a six-month thematic semester on *Computational Mathematics in Emerging Applications* at the Centre de recherches mathématiques (CRM) emphasized the increasing interdependency of using computational tools and engaging in the modelling, analysis, and solving of mathematical problems in applications. A report produced five years earlier, by the European Mathematical Society, also summarized the computationally-inspired change taking place in mathematics: "*Together with theory and experimentation, a third pillar of scientific inquiry of complex systems has emerged in the form of a combination of modelling, simulation, optimization and visualization*" (2011, p. 2).

In short, when engaging in our research, we look for the most effective tool(s) to assist us in advancing our work and our thoughts, whether it is pencil and paper, or some piece of software (e.g., Maple or SAS). If the tool turns out to be digital, we may not only interact with it as an integral part of our development of mathematics (as is the case when simulating or visualizing), but we may also do some programming to alter or create a tool that is more suitable for our specific questions – this in passing answers the "should we", doesn't it? Now let's continue with "how could we".

Reflecting on our own use of computational thinking has another purpose. A recent nationwide survey (N=302) found that the use of Computer Algebra Systems in research was the most significant factor for a Canadian mathematician to integrate it in his/her teaching (Buteau et al., 2014). And a similar international study, including the USA, England, and Hungary, led to the same result (Lavicza, 2008). If we assume this holds for other digital tools, then it could be important for a mathematics department seeking to integrate computational thinking to consider their professors' engagement in such thinking in their own research work.

But then, how could computational thinking be *integrated* in our undergraduate mathematics classrooms? In a recent work, Weintrop et al. (2016) enumerate different kinds of engagement that could be implemented in mathematics and science classrooms under the name of computational thinking. They categorize the so-called "practices" in relation to *Data, Modeling and Simulation*, *Computational Problem Solving*, and *Systems Thinking* (see Figure 1). The resulting framework is rooted in interviews the researchers conducted with scientists and mathematicians, as well as a comprehensive literature review. In Broley, Buteau, and Muller (2017), we provided further support of the framework by exemplifying each of the four categories of practices through specific mathematicians' research projects (originated from Broley's 2015 thesis) and undergraduate mathematics student assignments (originated from the Mathematics Integrated with Computers and Applications (MICA) courses at Brock University; Ben El-Mechaiekh, Buteau, & Ralph, 2007; Buteau, Muller, & Ralph, 2015).

Data Practices
Collecting data
Creating data
Manipulating data
Analyzing data
Visualizing data

Modeling & Simulation Practices
Using computational models to understand a concept
Using computational models to find and test solutions
Assessing computational models
Designing computational models
Constructing computational models

Computational Problem Solving Practices
Preparing problems for computational solutions
Programming
Choosing effective computational tools
Assessing different approaches / Solutions to a problem
Developing modular computational solutions
Creating computational abstractions
Troubleshooting and debugging

System Thinking Practices
Investigating a complex system as a whole
Understanding the relationships within a system
Thinking in levels
Communicating information about a system
Defining systems and managing complexity

Figure 1. Computational thinking practices in mathematics and science classrooms (Weintrop et al., 2016, p. 135).

The work of Weintrop et al. (2016) – Figure 1 – thus provides us with an idea of how computational thinking could be implemented in our undergraduate mathematics classrooms. Of course, the *actual* details of the implementation could vary greatly, especially when it comes to the degree to which students are asked to engage in the computational thinking themselves (Broley, Caron, & Saint-Aubin, forthcoming). It is our view that for computational thinking to be truly "integrated", we, as departments of mathematics, must also integrate it in our programs, i.e., in our departmental policies.

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Transitioning with Education Notes

John McLoughlin

Thank you Jennifer Hyndman

It was June of 2009 when I agreed to assume the editorial role with *Education Notes*. Ed Barbeau had offered his support in the transition and there were several months ahead before officially commencing in 2010. It was then that I made one of the most significant phone calls of my professional career.

Let me set the context. *Sharing Mathematics: A tribute to Jim Totten* was a conference held in Kamloops in May 2009 honouring our friend and colleague Jim Totten. The event featured several presentations and resulted with a collection of proceedings. One of the keynote presentations was by Jennifer Hyndman. The talk, *Hands-free Teaching*, subsequently appeared in the October 2009 issue of Education Notes (Volume 41, #6). The timing of our reconnection in Kamloops after a significant gap (since serving identical terms on the CMS Education Committee through 2001) seemed fortuitous. I decided to call Jennifer to suggest the idea of co-editing *Education Notes* for the next five years. I was delighted when Jennifer agreed to this and the two of us commenced as co-editors in January 2010 for what would become eight years. Curiously it was only months later that Jennifer Hyndman was receiving the CMS Excellence in Teaching Award at the summer meeting in Fredericton.

Jennifer is an excellent teacher and her written contributions to *Education Notes* are worth reading or revisiting for those interested in teaching. One of those contributions in October 2015 (Volume 47, #5) was entitled *Who are our teachers?* Jennifer is a dancer and her context for writing that piece grew out of an interplay of perceptions in mathematics and dance, as noted in her preface to the piece.

As I write this article I am recovering from the second day of a five day intensive dance camp for adults. I have just experienced being a student with four different teachers covering six different styles of movement. These teachers are at four different points in their careers but they all provided amazing experiences for the dancers. My development as a teacher has been strongly influenced by my experiences in the dance classroom and my growth continues to happen as I watch what the teachers do and how the students respond, and how I respond. I then translate that into how I see mathematics teaching and our students' experiences.

The high quality of Jennifer's writing, mathematics, and teaching combined with her personable professionalism made it truly an honour to work alongside her in this role over the past eight years. Jennifer made several written contributions though she also motivated others to write for *Education Notes*. Several of these grew out of her personal connections with people in the mathematical and/or scientific communities. Others involved outreach initiatives – one of the themes explored over our eight-year tenure. On that note, I want to say, "Thank you Jennifer for your commitment and collaboration. It has truly been a pleasure working with you."

What is ahead?

A new five-year term of the Education Notes will begin with the first 2018 issue. I welcome Kseniya Garaschuk aboard as the co-editor. One of the focal points of attention for *Education Notes* will be the first year experience and mathematical issues around that initial year of tertiary mathematics. Of course, contributions of articles and ideas from the community at large are welcomed on this subject or other topics.

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