STAGES OF STUDENTS' INSTRUMENTAL GENESIS OF PROGRAMMING FOR MATHEMATICAL INVESTIGATIONSand here

Chantal Buteau[1], Eric Muller[1], Joyce Mgombelo[1], & Ana Isabel Sacristán[2]

[1]Brock University, [2]Cinvestav-IPN.

Introduction

With the growing integration of programming in our mathematics classrooms, we see a crucial need to understand how (undergraduate) students may come to appropriate programming as an instrument (Rabardel, 1995/2002) for 'authentic' mathematical investigations, i.e., complete programming-based mathematical investigations "as mathematicians would do" (cf. Weintrop et al., 2016). We are proposing to extend the instrumental integration model (Assude, 2007) to describe four stages of students' appropriation process (i.e., instrumental genesis)

This poster draws from:

• 5-years research study, funded by the Canadian Social Sciences and Humanities Research Council (SSHRC), that seeks to examine how postsecondary mathematics students learn to use programming as a computational thinking instrument for mathematics.

• a naturalistic case study takes place in a sequence of three programming-based mathematics courses implemented in the mathematics department at Brock University (Canada) since 2001, where postsecondary students majoring in mathematics and future mathematics teachers learn to design, program, and use interactive computer environments to investigate mathematics conjectures, concepts, theorems, or real-world applications

•Data collected mainly through four student projects, semi-structured individual interviews and lab reflections

Theoretical Framework

The instrumental approach -Distinguishes between an artefact – a physical or non-physical object that is used to achieve a given task – and an instrument (psychological construct) which emerges when there is a meaningful relationship between the artefact and the user for a specific type of task (Rabardel, 2002).

The process by which a subject transforms an artifact into an instrument through schemes is called instrumental genesis which is a twofold process:

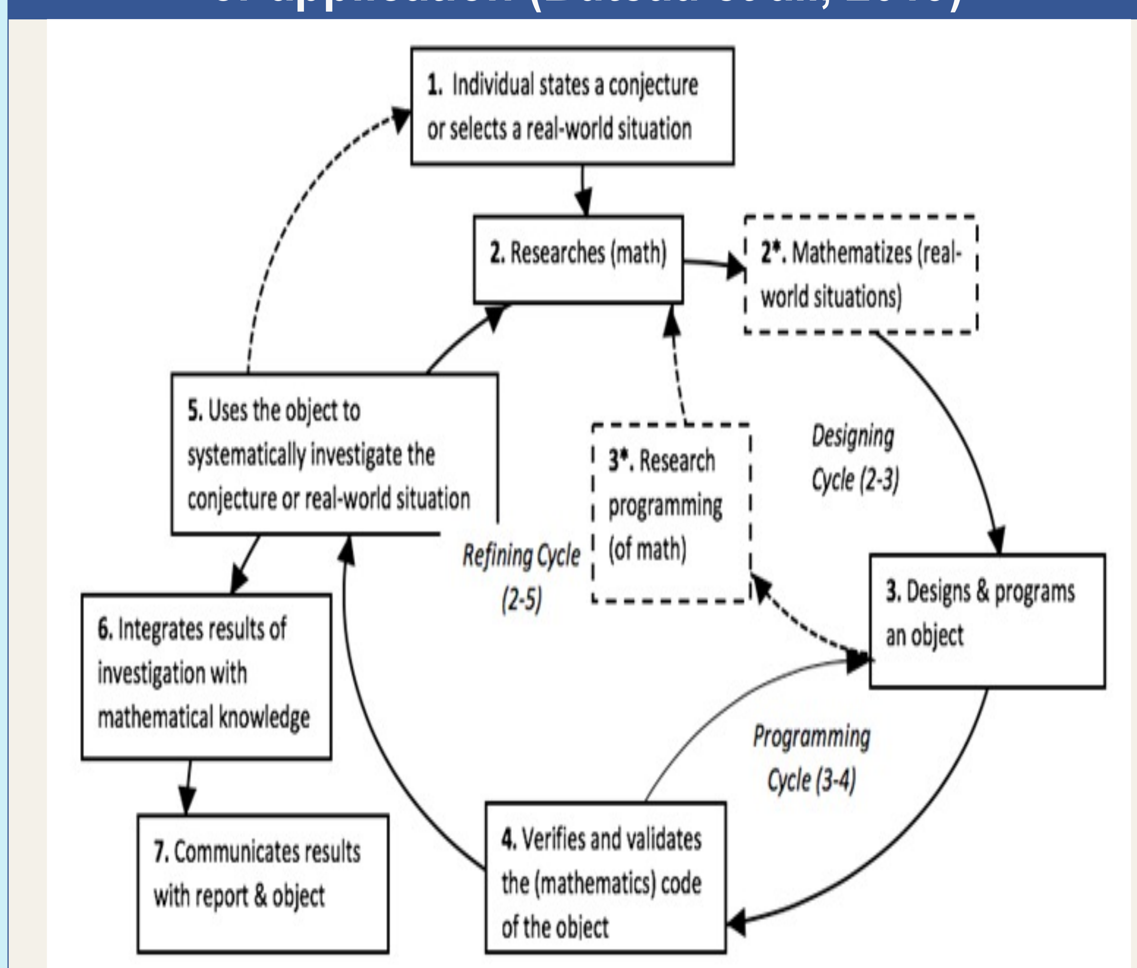
• instrumentalization –directed towards the artefact– the subject develops usage schemes, that are schemes oriented towards the management of the artifact;

•instrumentation –directed towards the user -the subject develops instrumented action schemes, that are schemes "oriented to the carrying out of specific tasks (Trouche, 2004,).

Learning – Legitimate Peripheral Participation- Viewed as situated activity – a process of legitimate peripheral participation where learners participate in communities of practitioners, and that the mastery of knowledge requires newcomers to move toward full participation in the sociocultural practices of a community (Lave and Wenger, 1991)

Focus on how students (newcomers) engage in computational thinking for mathematics as mathematicians (elders) would do (Weintrop et al., 2016)

Development process model of a student engaging in programming for a mathematical investigation or application (Buteau et al., 2019)

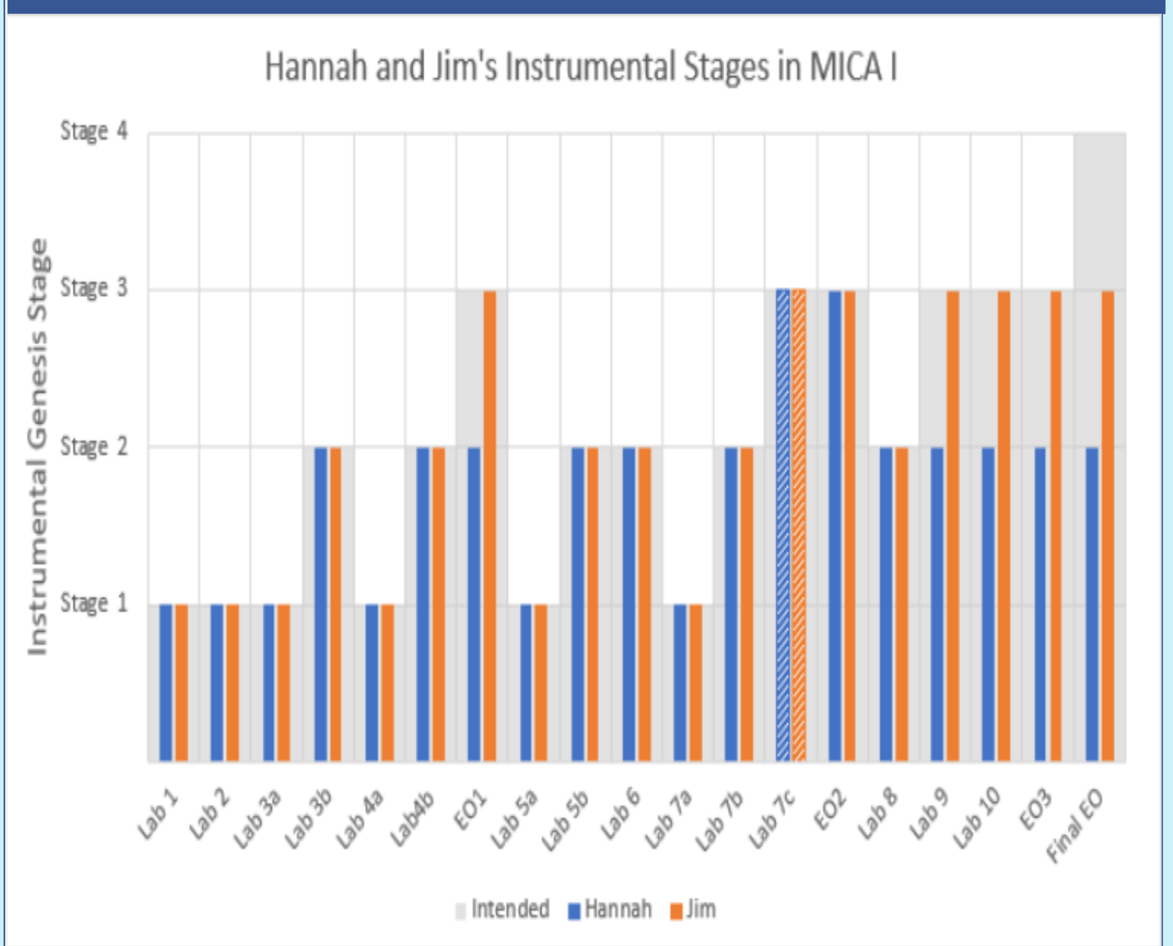


Stages of Instrumental Genesis for using Programming for Mathematical Investigations

Instrumental Stages	Teachers' Instrumental Integration Stages (Assude, 2009)	Students' Instrumental Genesis Stages of Using Programming for Authentic Mathematical Investigations
Stage 1. instrumental initiation	"pupils don't know the software and are initiated into Cabri tasks. The teacher's aim is mainly that the pupils learn how to use the software (pupils must learn **some IK**). The relation between IK and MK is minimal." (p.1342)	The student learns computational concepts articulated in a programming language and the management of the IDE, e.g. through programming exercises. This is associated to step 3 in the DP model. The student mainly develops **usage p-schemes**; e.g. the *scheme of coding a loop* or the *scheme of compiling the program*.
Stage 2. instrumental exploration	"pupils do not know the software and are going to explore it through mathematical tasks. The teacher aims at improving both IK and MK. The **relation between IK and MK can vary** from the minimum to the maximum according to the mathematical task. This mode can evolve to the instrumental symbiosis." (p.1342)	The student furthers his/her learning of computational concepts and learns computational practices (Brennan & Resnick, 2012) in the context of mathematics, e.g. through programming-based mathematics problems. This is associated to steps 3-4 and programming cycle in the DP model. The student engages in developing instrumented action **p-schemes** – e.g. the *scheme of code remixing*– as well as some **p+m-schemes** – e.g. the *scheme of* articulating *in the programming language a mathematical process*.
Stage 3. instrumental reinforcement	"pupils are already used to the software but they are confronted with instrumental difficulties while dealing with a mathematical task. The teacher will give them instrumental information. The teacher's aim is improving mathematical knowledge. The **relation between MK and IK is maximal** because IK is required to achieve the mathematical task." (p.1342)	The student learns computational practices and perspectives for mathematics (Weintrop et al., 2016), e.g. through authentic mathematical investigation projects. This is associated to mobilizing and developing schemes throughout the DP model. The student engages in furthering his/her instrumented action **p-schemes** and developing new or further instrumented action **p+m-schemes** as well as **m-schemes** – e.g. the *scheme of interpreting the program mathematics output.*
Stage 4. ***instrumental symbiosis***	"pupils have already used the software and they are confronted with mathematical tasks which allow them to **improve both their IK and MK because they are connected**. The relation between IK and MK is maximal: each one allows the other to increase and the connection between paper-pencil work and Cabri work is strong." (p.1342)	The student has sufficiently developed computational practices and perspectives enabling him/her to engage in CT for mathematics work (Weintrop et al., 2016). The student **can with ease mobilize and further develop schemes needed** for his/her self-undertaken authentic mathematical investigation projects. In other words, the students has appropriated programming as a CT-instrument for mathematics. In particular, this includes the p+m-scheme of identifying when a computational approach is relevant (i.e., an added-value over e.g. a paper & pencil approach) for the selected math investigation – as computational perspective.

- We associate Assude's knowledge indicator -Instrumental Knowledge (IK), Mathematical Knowledge (MK), IK/MK - to the development or mobilization of programming schemes (p-schemes), mathematics schemes (m-schemes), and p+m-schemes, respectively (see blue highlights).
- We associate Assude's type of task indicator (math, Cabri) to either programming exercises or to mathematics problems or projects, respectively (see red highlights).

Evolving instrumental stages throughout MICA I course



Summary of the seemingly evolving instrumental stages in which Hannah (in blue) and Jim (in orange) have been functioning throughout the different activities in the MICA I course. Letter numbering in the lab sessions indicates that there was more than one type of tasks presented to students.

The grey bars represent the intended instrumental stage based on the type of tasks. The dash bars at Lab7c indicate lack of evidence to evaluate the participant's instrumental stage.

Concluding notes

The instrumental approach provides us with a way of describing a student's learning in terms of scheme development for using a technology for a given type of mathematical task. The approach:has been applied to analyse a student's engagement often for a focussed type of task, leading to discussing their development of one or two schemes (e.g. Trouche, 2004; Drijvers, 2016). In this poster, we proposed an approach to extend this method when the type of tasks is an authentic programming-based mathematic investigation project that involves one core scheme comprised of a complex web of p-, p+m-, and m-schemes.

CTmathU Computational Thinking in University Mathematics

Social Sciences and Humanities Research Council of Canada | Conseil de recherches en sciences humaines du Canada

Canada

Brock University

Illustrating students' instrumental stages

Jim (pseudonym) is a first-year student in the Bachelor of Science in Mathematics program. His mindset with regard to mathematics and technology was very positive prior to entering the course. Formally, he had very limited knowledge of computer programming, except for some exposure in his youth with LOGO programming.

Hannah (pseudonym) is a mathematics and computer science co-major, and thus had programming experience (in Java) prior to her MICA I course.

Instrumental initiation (Stage 1)

Lab 3 aims in part at students learning about (usage p-scheme of writing) loops and conditional controls through programming exercises from the course programming textbook.

Jim mentions about adapting his already existing usage p-scheme of writing loop and conditional controls to the specific vb.net syntax, and expanding it to interrupt loops when needed:

"What I learned today mostly came down to new syntax to work with, such as how to exit a loop." (Jim.Lab3)

Hannah suggests adapting her already existing usage p-scheme of writing loop and conditional controls to the specific vb.net syntax, and expanding it to multiple loops and to interrupt loops when needed:

"In this lab I learned how to construct multiple loops and how to break out of them as soon as the conditions are met, which will definitely be helpful in the assignment." (Hannah.Lab3)

Instrumental Exploration (Stage 2)

Lab 3 also aimed at applying these computational concepts in the context of a mathematics problem, namely to create a program for checking the primality of any given positive integer.

Jim seems to suggest that he has started developing his p+m-scheme of articulating a mathematics process in vb.net as he points out the parallel of the hand-calculation method and the algorithm of the code:

"[I] do believe that, assuming I knew how to use the Mod command, I would have (given time) been able to make something resembling it from scratch, as the logic of how it searches for primes has already been touched upon in class." (Jim.Lab3)

Hannah's take on the articulation of the mathematics process, she claims she could have reproduced the code from scratch, but omits to justify why or how, which raises doubts that turn out to be confirmed later in the analysis:

"While the prime program was straightforward, and I would've been able to make it on my own, it would not have been as efficient as the lab version." (Hannah.Lab3)

Instrumental Reinforcement (Stage 3)

Lab 9 aimed at guiding students to use programming in a short authentic mathematical investigation project, namely to create a program that will allow them to systematically investigate the dynamical system based on the logistic function, a mathematics topic covered during lecture.

Jim seems to indicate having grasped, as a whole, the guided design of the mathematics exploration of the dynamical system investigation:

"I think the best way to describe my feelings would be the phrase: 'oh, ok, that's cool'. I thought it was neat how quickly programming in vb.net jumped from simple coding concepts to actually constructing visual output" (Jim.Lab9).

Hannah seems to have faced great difficulty in making sense of the program design for the aim of exploring the dynamical system; she says:

"This was a frustrating lab because I was struggling with the math understanding the math concepts and even though the programming itself was not hard, not knowing why I was doing something and how it should look like left me confused." (Hannah.Lab9)

References

Assude, T. (2007). Teachers' practices and degree of ICT integration. In D. Pitta-Pantazi & G. N. Philippou (Eds.), Proc. 5th Cong. of the European Society for Research in Mathematics Education (pp. 1339-1348). Larnaka, Cyprus: University of Cyprus.

Rabardel, P. (1995/2002). Les hommes et les technologies; approche cognitives des instruments contemporains. Paris, France: Armand Colin.

Trouche, L. (2004). Environnements informatisés et mathématiques: Quels usages pour quels apprentissages?Educational Studies in Mathematics, 55, 181–197

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. Journal for Science Education and Technology, 25, 127–147.