Effectiveness of a Project-Based Approach to Integrating Computing in Mathematics

Laura Broley
Brock University, Canada

Eunice Ablorh
Brock University, Canada

Chantal Buteau
Brock University, Canada

Joyce Mgombelo
Brock University, Canada

Eric Muller
Brock University, Canada

In this naturalistic study, we examine students' learning as they engage in programming for mathematics investigations through a project-based approach. We focus on undergraduate mathematics students' (N=41) engagement in a sequence of 14 programming-based math investigation projects, with data primarily collected through online questionnaires. Results suggest that students learn the most when they engage in projects they are passionate about. Results also provide empirical evidence supporting the effectiveness of a project-based approach by demonstrating the potential richness of students' learning in projects (e.g., learning general ways of doing and succeeding, in addition to specific mathematical knowledge).

Keywords: project-based learning, computing, student learning, mathematics investigation, operational and predicative knowledge

There are different approaches to integrating programming – or more broadly, computing – in undergraduate mathematics education, including: as a required skill (e.g., a computer science course requirement), within specific courses (e.g., modeling, numerical analysis), or through a more integrated approach (e.g., throughout a program or in a sequence of specially-designed courses). For instance, a survey of 46 mathematics departments in the U.K. found that 89% of undergraduate mathematics programs teach programming to all students, most commonly in numerical analysis or statistics (Sangwin & O'Toole, 2017). In comparison, at Manchester Metropolitan University in the U.K. (Lynch, 2020), University of Oslo in Norway (Malthe-Sørenssen et al., 2015), and Carroll College in the United States (Cline et al., 2020), programming is integrated across the undergraduate mathematics curriculum as a learning and/or problem-solving tool. In the Canadian context, Brock University's Department of Mathematics and Statistics has integrated programming since 2001 in a sequence of three specially designed project-based courses called Mathematics Integrated with Computers and Applications (MICA) I, II, and III (Buteau, Muller, & Ralph, 2015).

In a recent "Call for Research that Explores Relationships between Computing and Mathematical Thinking and Activity in RUME," Lockwood and Mørken (2021) suggest that different approaches provide "opportunities for systematically studying different ways for [the] integration [of computing] to occur" (p. 6). They point out, in particular: "there are an increasing number of examples of meaningfully-integrated programs across the world, and *the RUME community can explore what kinds of programs are effective and why*" (ibid., our emphasis). Four potential research foci are identified by the researchers, one of which is teaching. On page 7 of the call, potential research questions are proposed, including (our emphasis):

1. How should computing be introduced and taught in postsecondary mathematics classrooms and how might we design *effective tasks and curricular materials* to integrate computing into postsecondary mathematics classrooms?
2. What are *effective (or ineffective) program, department, and institution-level models* for integrating computing into mathematics classrooms?

The study we present is connected to the above call for research and potential questions.

We focus on the effectiveness of a particular kind of task and program model: those that are situated within or based on projects. There are some studies reporting on the effectiveness of project-based learning (PBL) in post-secondary STEM courses (Ralph, 2015; e.g., in chemistry courses that incorporate the use of computerized models, Barak & Dori, 2005). Nevertheless, a literature review on PBL in K-20 mathematics education (Jacques, 2017) highlights a lack of research and mixed results on the topic, concluding that "we cannot say at this time if PBL is or is not an effective approach" (p. 431).

In our 5-year research, we are interested in examining the effectiveness of teaching and learning environments such as in the MICA courses mentioned above. These courses use a PBL approach, engaging math majors and future math teachers in a sequence of 14 programming-based mathematics investigation projects (see Table 1 in Buteau, Muller, & Ralph, 2015 for examples of the 14 projects). Previous work has reported on the potential effectiveness of this sequence, based on task analyses and reflections from a few students (cf. Buteau et al., 2016). In this paper, we further investigate the (comparative) effectiveness of the MICA projects, as reported by a larger group (N=41) of students, based on their responses to a questionnaire.

Theoretical Framework

The origins of PBL can be traced to first century philosophers such as Aristotle, who believed that humans mainly learn by doing. More recently, theorists connected to the establishment of PBL (e.g., John Dewey) were inspired by an apparent gap between what students learn in school and the skills and attitudes they need to succeed in a constantly changing world. Building on Dewey's work, Kilpatrick (1921) defined a "project" as any unit of experience dominated by a purpose, which guides its process and drives its attainment. Other researchers have since provided a more elaborated definition, suggesting that projects should: (a) include complex tasks based on challenging questions or problems that involve students in design, problem-solving, decision making, or investigative activities; (b) give students the opportunity to work relatively autonomously over extended periods of time; and (c) culminate in realistic products or presentations (Jones, et al., 1997; Thomas, et al.,1999). Several other defining features of PBL can be found in the literature: e.g., (d) authentic content and assessment; and (e) teacher facilitation but not direction (Moursund, 1999).

The constructionist paradigm (Papert & Harel, 1991) embodies a particular kind of PBL approach in which students consciously and actively engage in constructing (e.g., through programming) tangible and shareable objects. Resnick's (2014) "4 P's" – projects, peers, passion, and play – describe some of the key instructional elements that can support an effective constructionist approach. He argues, for instance, that "when people work on projects they care about [i.e., that they are passionate about], they work longer and harder, persist in the face of challenges, and learn more in the process" (ibid., p. 1).

Though not explicitly intended, the MICA courses have been found to be intrinsically constructionist (Buteau, Muller, & Marshall, 2015). Also, the courses align with the features of PBL described above: MICA "projects" involve students in using programming for authentic pure and applied mathematics investigations ((a)/(d)); which are worked on autonomously, that is, facilitated rather than directed by instructors ((b)/(e)); and which culminate in useful computer environments and realistic project reports (c). Recall that the sequence of 14 MICA projects occurs over 3 courses: MICA I (Projects 1-4), MICA II (Projects 5-9), and MICA III (Projects 10-14). It is important to note that the last project in each course differs from the rest in that students can select a topic that interests them (all other projects concern topics that are specified by the instructor of the course). For instance, mathematics majors may decide to construct a program for investigating a problem or an area of research they find interesting; in a similar vein, mathematics teacher candidates may decide to program a "learning object" (Muller et al., 2009), i.e., a step-by-step guided learning of a school math concept, which may be relevant to their future profession (see Brock University, n.d., for some example projects). Inspired by Resnick (2014), we call these end-of-term projects "passion projects."

In this paper, we look at the "effectiveness" of projects in a PBL approach in terms of "student learning." Since our work focuses on a certain kind of PBL – where students are learning to use a particular digital tool (a programming language) – we further frame "student learning" using the notion of instrumental genesis (Guin et al., 2005; as described in Buteau et al., 2019). In mathematics education, instrumental genesis has been conceptualized as a complex process involving the intertwinement of learning techniques for using a digital tool and learning specific mathematics concepts (Artigue, 2002). For instance, the second MICA project in the sequence prompts students to implement an RSA algorithm to encode and decode messages. Students typically learn how to use functions and modules in order to program the algorithm, and they also learn the specific mathematics concepts underlying RSA encryption (e.g., modular arithmetic, Euclid's algorithm for finding the greatest common divisor, finding powers and inverses in Z_n, …). To elaborate further on these two kinds of learning, Vergnaud (2009) offers a conceptualization of knowledge that distinguishes between operational knowledge (which provides means to do and succeed) and predicative knowledge (which consists of means to express ideas in words or symbols). Although both types of knowledge may be involved in PBL, early proponents of the approach seemed to be trying to shift from the focus of math education on predicative knowledge, to also include an appropriate emphasis on operational knowledge.

In light of the above theoretical framework, we pose two research questions:

1. What kind of programming-based math investigation projects are most effective?
2. In what ways are they effective (e.g., what kind of knowledge is learned)?

In this paper, we address these questions from the student's point of view.

Methodology

The study we present is part of a larger 5-year (2017-22) iterative design research focusing on the learning and teaching of programming for authentic pure and applied math investigations. This is a naturalistic research contextualized in the MICA courses, which are semester-long mathematics courses including 2-hour lectures and 2-hour labs each week. Part of the research closely follows some MICA students over the MICA I, II, and III courses, using individual semi-structured interviews related to their engagement in each of the 14 programming-based mathematics projects (P1, …, P14), as well as their lab reflections and project reports. To complement this in-depth study of individual students' learning, in each MICA course students are invited to voluntarily participate in pre and post anonymous[1] questionnaires.

The questionnaire data has been collected so far in Years 2-4 of the research. Questions in the pre-questionnaire differ from those in the post- since the latter invites students to reflect on their learning during the course. The questionnaires feature several sections, such as participants' demographics and participants' perceptions of: the importance/usefulness of digital technology (including programming), their own knowledge/confidence level in programming, what it means to learn/do math, and the course (the assignments, the teaching, and their learning).

[1] Students who participate in the in-depth study are not anonymous.

This paper focuses on a post-questionnaire question that contains two parts. In the first part, participants are asked to indicate which project they learned the most from, from all the projects in the MICA courses they had taken (for instance, a MICA II participant could select one project from either MICA I or II, i.e., from among P1 to P9). In the second part, participants are then asked to select a reason for the project they chose. Options are provided and participants can choose more than one option if they wish. The possible options are: (a) I learned a lot of new math; (b) I learned a real-world math application that really speaks to me; (c) I completed it all by myself without help; (d) It was challenging but I finally understood it; (e) I had to use new programming concepts or skills; (f) I discovered something I did not expect; and (g) Other.

41 students responded to the above question (24 from MICA I, 5 from MICA II, 12 from MICA III), which is not necessarily representative of the entire MICA student population (a limitation of our study). Responses were analysed using comparative bar charts and frequency tables. The first part of the questionnaire question was key to answering our first research question: When a participant indicated the project they learned the most from, we interpreted this as the "most effective" project from that participant's point of view, and we reflected on the kinds of projects that were selected. This reflection was further supported by the second part of the questionnaire question (students' reasons for their selected project), which was also key to answering our second research question (concerning the ways in which the projects are effective). As part of our analysis, we regrouped the above reasons into two categories based on whether they were indicative of learning predicative or operational knowledge (as defined in our Theoretical Framework). In the predicative knowledge category, we include (a), (b), and (f), which we see as reflecting project-specific learning of math concepts. In the operational knowledge category, we include (c), (d), and (e), which we see as reflecting learning of more general ways of doing and succeeding in using programming for math investigation. We note that each of the "Other" responses were reviewed to determine if they could be categorized as any of (a)-(f) (in which case they were recategorized and frequencies were adjusted accordingly).

Results and Discussion

We organize our results and discussion according to our two research questions.

The Most Effective Projects

Table 1 shows which programming-based mathematics investigation projects from among the sequence of 14 that participants selected as the ones they learned the most from. Note that we distinguish what we called "passion projects" with the notation "PP."

Table 1. The projects participants said they learned the most from.

Course	P1	P2	P3	PP4	P5	P6	P7	P8	PP9	P10	P11	P12	P13	PP14
MICA I	1	10	5	8	-	-	-	-	-	-	-	-	-	-
MICA II	0	0	0	1	0	0	0	1	3	-	-	-	-	-
MICA III	0	1	0	2	0	0	0	1	2	0	1	0	1	4
Total	**1**	**11**	**5**	**11**	**0**	**0**	**0**	**2**	**5**	**0**	**1**	**0**	**1**	**4**

We observe that students selected a variety of projects. Some concern pure mathematics (P1: conjectures about primes or hailstone sequences, P3: a discrete dynamical system, or P11: simulations related to Bertrand's paradox), while others concern applied mathematics (P2: RSA encryption, P8: battle simulations, or P13: randomness of DNA sequences). Some have a higher ceiling in terms of potential for discovery and investigation (e.g., P1, where students are invited to choose or formulate a mathematical conjecture to explore, vs. P2, where students apply the RSA method to encrypt and decrypt messages). Some projects also differ in the degree to which they are scaffolded by instructors (e.g., P2, where students are guided through some steps to construct their program, vs. PPs, where students must determine the design and use of their programs). Finally, the projects chosen by participants show up at different moments in the sequence (e.g., a MICA III participant selected P2, which occurs towards the beginning of the sequence; another selected P13, which occurs towards the end).

Among the projects that were selected, nearly half (20/41) were PPs, in which students choose the topic of their investigations. This provides empirical evidence in support of certain constructionist claims: e.g., that students learn best when they are working on projects that are meaningful to them (Papert, 1980), i.e., on topics that they are passionate about (Resnick, 2014). We also note that about half of the participants who selected PPs indicated their passion project from MICA I, evidencing the low-floor-high-ceiling affordance of programming for mathematics learning (Gadanidis, 2017): already in their first year, with minimal programming background, these students said that they engaged in meaningful learning in such a project.

When looking at MICA I participants only, P2 was the most selected (41.7%), even over the MICA I final project (PP4; 33.3%). At first, this seems to be a rather surprising result, for example, due to P2's lower ceiling in terms of potential for discovery and investigation (Buteau et al., 2018), especially when compared to PPs. We discuss this result more in the next section.

The Ways in Which the Projects are Effective

In selecting the reasons for the project they learned the most from, participants had the option to select more than one reason. Figure 1 shows the number of reasons selected by participants (left) and the percentage of participants who selected the different possible reasons (right).

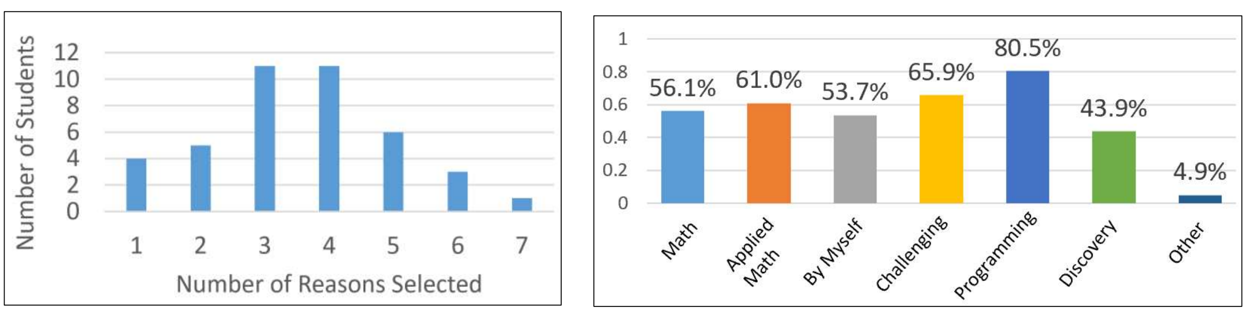

Figure 1. Number of reasons selected (left) and percentages of participants (N=41) selecting each reason (right).

A high percentage of participants (78%) selected at least 3 reasons. This highlights the richness of the students' learning experiences in their chosen projects.

"I had to use new programming concepts or skills" was selected by the highest percentage of students (80.5%). This shows that as students engage in these projects, they typically encounter avenues where in addition to applying their prior knowledge in programming, they also have to use new knowledge to create their program and complete their projects. Further, as seen in Figure 1 (right), at least half of participants selected that they persevered in the face of challenges (65.9%), learned a real-world math application that spoke to them (61.0%), learned lots of new math (56.1%), or completed the projects by themselves without help (53.7%). Also, close to half (43.9%) said they discovered something they did not expect. We argue that the prevalence of these different reasons, as well as the selection of multiple reasons mentioned above, provides evidence in support of the effectiveness of a PBL approach.

Only 2 students (4.9%) gave "Other" reasons for why they benefitted the most from the projects they chose (both of which were PPs). One student said, in addition to (a), (b), and (d): "I was paired with a cool partner and we worked well together." The other, who had designed and programmed a learning object, said, in addition to (c) and (e): "How to create a lesson plan which incorporates programming to teach mathematics to grade 9 students." These responses are linked to two characteristics of effective constructionist learning, as highlighted by Resnick (2014): the former to "peers" and the latter to "passion." It is notable that these two participants felt the need to specify these additional reasons for the effectiveness of their chosen projects.

As mentioned above, PPs and P2 were the most commonly selected projects by the participants. Participants' reasons for selecting these are depicted in Figure 2.

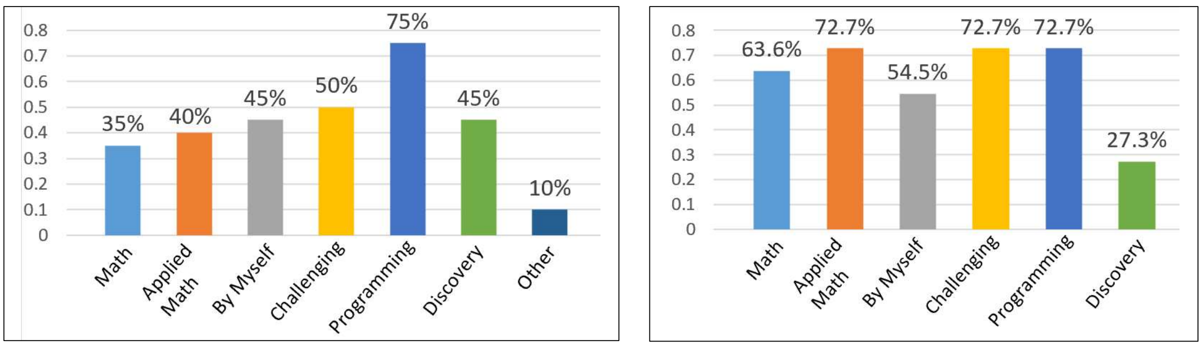

Figure 2. Percentages of participants who selected each reason for PPs (left; N=20) and P2 (right; N=11).

When just looking at PPs (Figure 2, left), we note that only two responses were selected by at least 50% of participants: "I had to use new programming concepts and skills" (75%) and "It was challenging but I finally understood it" (50%). In the case of PPs, students select their mathematics topic, which can naturally lead to new demanding needs in terms of the programming (as opposed to just using the programming learned during prior MICA projects). These results suggest that students' learning experiences in PPs may be more pointed.

In comparison, 5 reasons were selected by at least 50% of participants who chose P2, suggesting that P2 provides an opportunity for students to experience multiple facets of learning. This is likely linked to the position of P2 in the sequence of 14 MICA projects. For most students, P2 is the first project where they must program a more elaborate mathematical process, in addition to using new programming concepts and skills. This represents a steep learning curve for students: overcoming it, possibly almost all by themselves, would constitute a major accomplishment. The fact that students indicate learning a real-world application that speaks to them is not surprising: P2 concerns RSA encryption, a topic that relates to the digital world in which the students live. Interestingly, the other reason ("I discovered something I did not expect") was selected by less than 30% of participants, which seems to indicate that the low ceiling for discovery in P2 (as speculated in Buteau et al., 2018) is reflected in students' perceptions. We also point out that students coming from high school may not be used to investigating conjectures: therefore, after a more unsettling experience with P1, students may feel more comfortable with a more directed project like P2, where they can follow instructions and easily see the progress of their engagement. Anecdotally speaking, we have heard many other students state that P2 was a particularly memorable project.

Operational and predicative knowledge. In this subsection, we report on the kind of knowledge students indicated they learned in their chosen projects, based on our regrouping of reasons discussed in the Methodology. Table 2 summarizes the results.

Table 2. The kind of knowledge students learned in their chosen projects.

Projects	Operational & Predicative	Only Operational	Only Predicative
All	30	8	3
PP	13	5	2
P2	9	2	0

It appears that over 70% of participants learned both operational and predicative knowledge in the project they learned the most from. This reflects the different kinds of knowledge required in using programming for mathematics like mathematicians do (Broley, 2015). Also, over 90% of participants indicated learning operational knowledge. This aligns with proponents of PBL (e.g., Kilpatrick, 1921), who talk about the approach as an attempt to engage students in learning the general processes and attitudes involved in (mathematical) problem solving, in addition to learning specific (math) content. One could wonder about the 7.3% of participants who indicated learning solely predicative knowledge in the project they learned the most from. We note that these three participants were future math teachers. Two selected PPs, which could have been a learning object connected to their future career. If they did it in pairs, it is possible that the participant did not lead the coding of the object, and thus did not feel as though they gained operational knowledge.

Results for P2 in Table 2 align with the above results. In comparison, Table 2 offers a different perspective on the richness of learning experiences in PPs. Despite only a few reasons being identified by high percentages of participants (Figure 1), Table 2 suggests that many (65%) gained both operational and predicative knowledge.

Conclusion

In this paper, we answered Lockwood and Mørken's (2021) call for research by exploring the effectiveness of one approach to integrating computing in undergraduate mathematics: namely, PBL, as defined by general work in education (e.g., Jones, et al., 1997; Moursund, 1999; Thomas, et al.,1999) and the constructionist paradigm (e.g., Papert, 1980; Resnick, 2014). One main contribution of our study is that it addresses the gap of empirical evidence related to the effectiveness of such an approach (e.g., reporting on the different kinds of learning that may occur in programming-based mathematics investigation projects).

Our study was based on student (N=41) responses to an online questionnaire and was exploratory in nature. The theoretical framework and results inspire us to refine our questionnaire: in particular, to rework the reasons that students can choose to explain the project they learned the most from. There seem to be some reasons that could be revised (e.g., 'I learned a real-world math application that really spoke to me' could be divided into 'I learned a real-world math application' and 'I learned something that really spoke to me'). There also seems to be other reasons that are important to include (e.g., 'I worked with others'). As part of our larger 5-year research, we also have interview data, which could support us in digging deeper into the effectiveness of these projects and the types of learning that are involved (e.g., the more pointed yet significant learning that may be happening in passion projects, the rich learning in P2, or the combined learning of operational and predicative knowledge across the project sequence).

Acknowledgments

This work is funded by the Social Sciences and Humanities Research Council of Canada (#435-2017-0367) and received ethics clearance at Brock University (REB #17-088).

References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245–274.

Barak, M., & Dori, Y. J. (2005). Enhancing undergraduate students' chemistry understanding through project-based learning in an IT environment. *Science Education*, *89*(1), 117–139.

Brock University. (n.d.). *MICA – Mathematics Integrated with Computers and Applications.* www.brocku.ca/mathematics/studentprojects

Broley, L. (2015). *La programmation informatique dans la recherche et la formation en mathématiques au niveau universitaire* [Master's thesis, Université de Montréal]. Papyrus: Institutional Repository. https://papyrus. bib.umontreal.ca/xmlui/handle/1866/12574.

Buteau, C., Gueudet, G., Muller, E., Mgombelo, J. & Sacristán, A. I. (2019). University students turning computer programming into an instrument for "authentic" mathematical work. *International Journal of Mathematical Education in Science and Technology*, *57*(7), 1020–1041.

Buteau, C., Muller, E., & Marshall, N. (2015). When a university mathematics department adopted core mathematics courses of an unintentionally constructionist nature: Really? *Digital Experiences in Mathematics Education*, *1*(2/3), 133–155.

Buteau, C., Muller, E., Marshall, N., Sacristán, A., & Mgombelo, J. (2016). Undergraduate mathematics students appropriating programming as a tool for modelling, simulation, and visualization: A case study. *Digital Experiences in Mathematics Education*, *2*(2), 142–166.

Buteau, C., Muller, E., Mgombelo, J., Rafiepour, A., & Sacristán, A. I. (2018, September). *Features of 'authentic' programming-based mathematical tasks* [Poster presentation]. Mathematics Education in the Digital Age, Copenhagen, Denmark. https://ctuniversitymath.files.wordpress.com/2021/09/meda-2018-poster-buteau-et-al..pptx

Buteau, C., Muller, E., & Ralph, B. (2015). *Integration of programming in the undergraduate mathematics program at Brock University* [Paper presentation]. Math + Coding Symposium, London, ON, Canada. https://researchideas.ca/coding/docs/ButeauMullerRalph-Coding+MathProceedings-FINAL.pdf

Cline, K., Fasteen, J., Francis, A., Sullivan E., & Wendt T. (2020). Integrating programming across the undergraduate mathematics curriculum. *PRIMUS*, *30*(7), 735–749.

Gadanidis, G. (2017). Five affordances of computational thinking to support elementary mathematics education. *Journal of Computers in Mathematics and Science Teaching*, *36*(2), 143–151.

Guin, D., Ruthven, K., & Trouche, L. (2005). *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument*. New York, NY: Springer.

Jacques, L. A. (2017). What does project-based learning (PBL) look like in the mathematics classroom? *American Journal of Educational Research, 5*(4), 428–433.

Jones, B. F., Rasmussen, C. M., & Moffitt, M. C. (1997). *Real-life problem solving.: A collaborative approach to interdisciplinary learning*. Washington, DC: American Psychological Association.

Kilpatrick, W. H. (1921). Dangers and difficulties of the project method and how to overcome them- A symposium. *Teachers College Record, 22*(4), 283–288

Lockwood, E., & Mørken, K. (2021). A call for research that explores relationships between computing and mathematical thinking and activity in RUME. *International Journal of Research in Undergraduate Mathematics Education*, 1–13.

Lynch, S. (2020). Programming in the mathematics curriculum at Manchester Metropolitan University. *MSOR Connections*, *18*(2), 5–12.

Malthe-Sørenssen, A., Hjorth-Jensen, M., Langtangen, H. P., & Mørken, K. (2015). Integration of calculations in physics teaching. *Uniped*, *38*, Article 6.

Moursund, D. G. (1999). *Project-based learning using information technology*. Eugene, OR: International society for technology in education.

Muller, E., Buteau, C., Ralph, W., & Mgombelo, J. (2009). Learning mathematics through the design and implementation of Exploratory and Learning Objects. *International Journal for Technology in Mathematics Education, 16*(2), 63–74.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. Basic Books.

Papert, S., & Harel, I. (1991). Situating constructionism. In I. Harel & S. Papert (Eds.), *Constructionism* (pp. 1–11). Ablex.

Ralph, R. A. (2015). Post secondary project-based learning in science, technology, engineering and mathematics. *Journal of Technology and Science Education, 6*(1), 26–35.

Resnick, M. (2014). *Give P's a chance: Projects, peers, passion, play* [Opening keynote]. Constructionism and Creativity conference, Vienna University of Technology, Vienna, Austria.

Sangwin, C. J., & O'Toole, C. (2017). Computer programming in the UK undergraduate mathematics curriculum. *International Journal of Mathematical Education in Science and Technology*, *48*(8), 1133–1152.

Thomas, J. W., Mergendoller, J. R., and Michaelson, A. (1999). *Project-based learning: A handbook for middle and high school teachers*. Novato, CA: The Buck Institute for Education.

Vergnaud, G. (2009). Colloquium de didactique des mathématiques. Activité, développement, représentation. In L. Coulange, & C. Hache (Eds.), *Actes du séminaire national de didactique des mathématiques* (pp. 7–29). IREM Paris 7 et ARDM.