Teaching in a Sustained Post-Secondary Constructionist Implementation of Computational Thinking for Mathematics

Chantal Buteau, *cbuteau@brocku.ca*
Brock University, Canada

Ana Isabel Sacristán, *asacrist@cinvestav.mx*
Cinvestav, Mexico

Eric Muller, *emuller@brocku.ca*
Brock University, Canada

Abstract

In this practice report, we reflect and discuss the roles of, and demands on university instructors in three undergraduate mathematics computer-based courses implemented since 2001 at Brock University, Canada. These are the *Mathematics Integrated with Computers and Applications (MICA) I, II, III* courses, in which instructors create an environment that supports students' constructionist learning experiences as they design, program, and use interactive environments (i.e., microworlds) to learn and do mathematics. Using Ruthven's (2009) model on the professional adaptation of classroom practice with technology, we feature constructionist characteristics of the course design highlighting the shift from traditional, instructionist pedagogy towards one of empowering students. Since there seem to be relatively few sustained implementations of microworlds in mathematics instruction (Healy & Kynigos, 2010), this report, grounded on a continuous practice of over 15 years, contributes to our understanding of roles and demands of "ordinary" instructors in the "real" classroom, who have aimed at creating an environment for supporting students' constructionist learning experiences. In particular, this report highlights the instructor's demanding role in these student-centred courses, more so since students select their own topics for their last project in lieu of final exam, thereby having the opportunity of it being meaningful to them.

Keywords

computational thinking; constructionism; mathematics; programming; microworlds; university; teachers

Introduction

In this practice report, we reflect on the roles of, and demands on university instructors in undergraduate mathematics computer-based courses – the *Mathematics Integrated with Computers and Applications* (MICA) *I-II-III* courses – carried out at Brock University, Canada (Ralph, 2001; Buteau, Muller, & Ralph, 2015). These courses follow the constructionist paradigm (Papert & Harel, 1991), requiring students to design and program computational objects for mathematical learning, and have had sustained implementation for over 15 years (Buteau, Muller & Marshall, 2015).

More specifically, at Brock University, mathematics majors and future mathematics teachers learn to design, program, and use interactive environments –called *Exploratory Objects*– for the investigation of mathematical concepts, conjectures, and theorems or real-world situations (Muller, Buteau, Ralph, & Mgombelo, 2009). During their first undergraduate years, students may enrol in a sequence of three MICA courses and create in total 14 Exploratory Objects as part of their course load (Buteau, Muller, Marshall, Sacristán, & Mgombelo, 2016). At the end of each term, students, individually or in groups of two or three, create an original Exploratory Object for which they select the topic (see MICA, 2018). As a result of a literature review study (Marshall & Buteau, 2014), we classified the development of Exploratory Objects as mathematical microworlds: they constitute open-ended exploratory computer activities (Edwards, 1995) where MICA students engage in computational thinking for mathematics (Buteau et al., 2016).

This paper provides a discussion, based on insightful reflections, on the “real” MICA classroom by particularly focusing on how the “ordinary” MICA instructors have created an environment that supports the students’ constructionist learning experiences. We are interested in discussing the constructionist experiences involved in the teaching of MICA courses. In the following, we first briefly lay out aspects of Constructionism, Computational Thinking and Microworlds that frame the pedagogical approach in the MICA classroom. Using Ruthven’s (2009) model of five key structuring features of school classroom practice, we then discuss demands on, and roles of instructors in these courses. This discussion focuses, first, on the overall courses and, second, on students’ final individual projects which are used in lieu of final exams. We end with a few concluding remarks.

Constructionism, Computational Thinking and Microworlds: A Theoretical Framework for MICA’s Pedagogical Approach in the Mathematics Classroom

As stated above, our MICA courses follow the constructionist paradigm; constructionism is defined by Papert and Harel (1991, p.1) in the following way:

> *Constructionism--the N word as opposed to the V word--shares constructivism's connotation of learning as ´building knowledge structures´ irrespective of the circumstances of the learning. It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe.*

Thus, the basic principles of the constructionist paradigm involve learning situations or environments that are student-centred, where students build or construct shareable objects that are somehow “tangible”. These situations usually involve open projects, often computer-based ones, in which learners engage with meaningful “powerful ideas” (Papert, 1980) within a social context of collaboration, discussion or interaction among peers.

One concept that is linked to constructionism, is that of exploratory computational environments known as microworlds. A microworld is defined by diSessa (2000) as

> *a type of computational document aimed at embedding important ideas in a form that students can readily explore. The best microworlds have an easy-to-understand set of operations that students can use to engage tasks of value to them, and in doing so, they come to understanding powerful underlying principles. (p. 47)*

For Papert (1980), microworlds involve objects “to think with” and “allow a human learner to exercise particular powerful ideas or intellectual skills” (p. 204) through exploration and discovery in a knowledge domain.

Two ways of describing microworlds are given by Edwards (1995): a “structural” definition which focuses on the design elements (e.g. collections of computational objects to model mathematical or physical properties of a domain; representations; and activities or challenges for students to explore in the domain); and a “functional” definition which highlights how students learn with microworlds, such as through the interaction between the student, the software, and the setting in which it is used.

Related to the latter, an important aspect for us is that related to the role of the teacher in constructionism: Papert (1980) stated that in a microworld situation “the relationship of the teacher to learner is very different: the teacher introduces the learner to the microworld in which discoveries will be made, rather than to the discovery itself” (p. 209); that is, the situation presented by the teacher is one to facilitate students to think like mathematicians rather than to teach them about mathematics.

Weintrop et al. (2016) remind us that “Papert (1996) was the first to use the term computational thinking to refer to the affordances of computational representations for expressing powerful ideas.” (p.130). These authors discuss extensively on how mathematicians and scientists have come to engage in computational thinking in their professional (including research) work, and argue that “the varied and applied use of computational thinking by experts in the field provides a roadmap for what computational thinking instruction should include in the classroom" (p. 128).

In the next sections we present the structure and functioning of our MICA courses, highlighting the constructionist aspects and role of the instructors.

University Instructors in MICA Courses: Roles & Demands

To provide a structure for our reflection and discussion on demands on, and roles of instructors in MICA courses, we use Ruthven's (2009) model of five key structuring features of school classroom practice to "*illuminate the professional adaptation which technology integration into classroom practice depends*" (p. 131).

Working environment

For MICA courses, instructors teach in regular lecture rooms where they mostly elaborate the mathematical content. They lead mathematics programming-based activities in computer laboratories (one computer per student). The course format is two hours per week for lectures and two hours per week for lab sessions. The class size is capped at 35 students per course section.

Resource system

Ruthven (2009) describes: "[t]he concept of 'resource system' focuses… on the combined operation of the mathematical tools and curriculum materials in classroom use, particularly on their compatibility and coherence of use, and on factors influencing this" (p.136). For their mathematics lectures, MICA I instructors use lecture notes which to date have been shared among instructors. In MICA II, instructors use lecture notes and possibly a mathematical modelling textbook depending on the instructor. For the lab sessions, programming software is used. Since MICA I students need to learn programming, the instructor uses a friendly textbook for the introduction of this technology (currently VB.NET in Visual Studio environment), in addition to lab activity guidelines (Ralph, 2017) that emphasize the connection of mathematics and programming (Buteau & Muller, 2014). In MICA II-III, instructors use only lab activity guidelines since students are then knowledgeable programmers. MICA instructors actually use programming in their own mathematical research, and therefore consider programming as an integral part in doing mathematics (i.e. they are engaging in computational-thinking-based mathematics research as described by Weintrop et al., 2016). In other words, because of their research practice, instructors naturally merge the mathematics and programming resources as a system. Individual instructors might need to learn the specific programming language. However, this is a not overwhelming particularly since within the lab sessions they can count on knowledgeable teaching assistants.

Activity format

Ruthven (2009) introduces the key structuring feature of activity format where "Classroom activity is organised around formats for action and interaction which frame the contributions of teacher and students to particular lesson segments (Burns & Anderson, 1987; Burns & Lash, 1986)." (p. 137).

Overall the MICA instructor needs to provide a learning environment in which students:

- design and program mathematics experiments/modeling/simulations with an appropriate interface in order to conduct an investigation;
- reflect on mathematical results and data in a written report.

(Buteau & Muller, 2010)

This constitutes what we call in this paper 'a microworld approach' to learn and do mathematics. In the first-year MICA I course, students learn the microworld approach, and in the upper-year MICA II-III courses, they apply it to broader and more sophisticated mathematics contexts (Buteau et al., 2016). The instructor in any MICA I – II - III provides an environment for students to experience constructionist learning: students learn by making and using these microworld projects. This includes a pedagogical approach where instructors guide students to carefully develop their visual interfaces to support their investigations (the construction is shareable). In fact, this overall aim for students to learn and apply the microworld approach grounds the MICA course design and pedagogy (Buteau et al., 2015), and leads to empowering students to work as mathematicians (Broley, Buteau, & Muller, 2017; Buteau et al., 2016).

In MICA I lectures, the instructor elaborates on the mathematical content, although it is not presented quite in a traditional manner. For example, the instructor thinks out loud as a ‘working mathematician’ to make transparent the messy development of mathematical ideas. Or the content may be presented through selected examples in order to prompt students to ask mathematics questions and state conjectures. This is a shift from a traditional, instructionist approach towards one of empowering students. For the instructor, it requires acceptance of a slower pace in terms of mathematics content covered and surrendering some control (i.e., s/he acts as facilitator, not lecturer). In MICA II-III, the instructor follows a more regular lecture format. However, due to the smaller size of classes and the inquiry component of the course, lectures are often more interactive including aspects of an inquiry-based mathematics classroom (Rasmussen & Kwon, 2007); e.g. with interaction between instructor-students or among students, at structured times.

In MICA lab sessions, the instructor usually gives overall guidelines to the whole class for a short period of time (2-15 minutes), then leads students to their individual work, and when necessary, provides opportunities for the whole class to discuss a situation encountered by one or more students. Teaching assistants assist the instructor with students’ individual work. This stresses the student-centred activity format of the lab sessions. Overall, and particularly in MICA I, instructors need sensitivity to individual students’ development of their instrumental genesis (Trouche, 2004) during their programming and in their involvement in the mathematics microworlds (Buteau & Muller, 2014), stressing again the student-centred characteristic of MICA courses.

Curriculum script

When preparing or teaching lessons on a topic, instructors use their professional knowledge that includes, in particular, “a loosely ordered model of relevant goals and actions which serves to guide their teaching of the topic” (Ruthven, 2009, p.138).

MICA courses are not mathematics content-driven: the microworld approach to learn and do mathematics defines the courses (Buteau et al., 2015). The overall content is not ‘traditional mathematics’, and the overall script had to be rethought since the technology impacts ‘what’ mathematics is covered in MICA courses (Buteau et al., 2016). In MICA I, the course content is driven by students needing to learn programming and the microworld approach: the mathematics content provides context in which students learn those (Buteau & Muller, 2014). In particular, the learning of programming technology for mathematics follows a back-and-forth instrumental orchestration (Trouche, 2004) model (Buteau & Muller, 2014). The instructors’ script also includes the component of debugging one’s own program and for students to appreciate the value of debugging. In MICA II-III, each instructor involves mathematics content relevant to the microworld approach (i.e., a computational thinking approach) according to his/her own, evolving mathematics interests. As such, the instructors integrate authentic programming-based mathematics tasks (Buteau, Mgombelo, Muller, Rafiepour & Sacristán, submitted) which, in particular, includes inquiry practices (i.e., involving *using* the microworld). We associate this with Wagh, Cook-Whitt and Wilensky’s (2017) position that “the constructionist approach of interacting with and manipulating program code of computational models can facilitate productive forms of [students’] engagement with inquiry-based science” (p.615).

Time economy

Because of the student-centered characteristic of the microworld approach defining the MICA courses, individual student guidance is required, and this means ‘time’. To respond to this characteristic, the department limits enrolment to 35 students per section and provides two teaching assistants for the lab sessions to assist the instructor.

As mentioned before, the instructors need to accept and adapt, due to the nature of the MICA courses, to a slower pace in terms of mathematics content covered in lectures. Also, instructors use programming in their own research and do not require additional time to learn the technology (except if there is a change in the programming language). However, when teaching the MICA courses for the first time, instructors need to prepare in terms of the change in the pedagogical paradigm required for the MICA implementation of the microworld approach (Papert, 1980; Muller et al., 2009). This is a shift in philosophy that is new to university mathematics instructors who normally mainly, if not strictly, concentrate on content. This shift requires time.

University instructors have a dual role for their departments as teachers and policy makers to create, delete, and modify courses in their curriculum (Barker et al., 2004). Because of the innovative nature at the time of the proposed course design, a professor in the department was awarded a course release to dedicate his time to design what became MICA courses (Buteau et al., 2015).

In the next section, we elaborate on the roles of and demands on the instructors required in the last part of the MICA courses for which the course format is significantly different.

Instructors Creating an Environment for Individual Projects Meaningful to Students: Roles & Demands

Each of the three MICA course terms culminates to a complete constructionist learning experience opportunity for every student. This is a very demanding and engaging exercise for both students and instructors.

Students, individually or in groups of two or three, create original microworld open projects for which they select their own topics. The MICA instructor encourages and insists that students select a topic of interest to them, i.e., something related to them, either an application of mathematics or a mathematical topic of interest. In other words, the instructor motivates students to engage in a project that is meaningful to them. For example, Matthew and Kylie wondered if it is better to walk or run in the rain (Figure 1, left) while Adam investigated the bounded area, as the exponent increases, of the iterative complex function defining the Mandelbrot set (Figure 1, right). See MICA (2018) for these and other examples. Since these original microworlds are meaningful to students and shareable constructions, we have observed that students show pride of, engagement in, and ownership of their own projects (Muller et al., 2009).

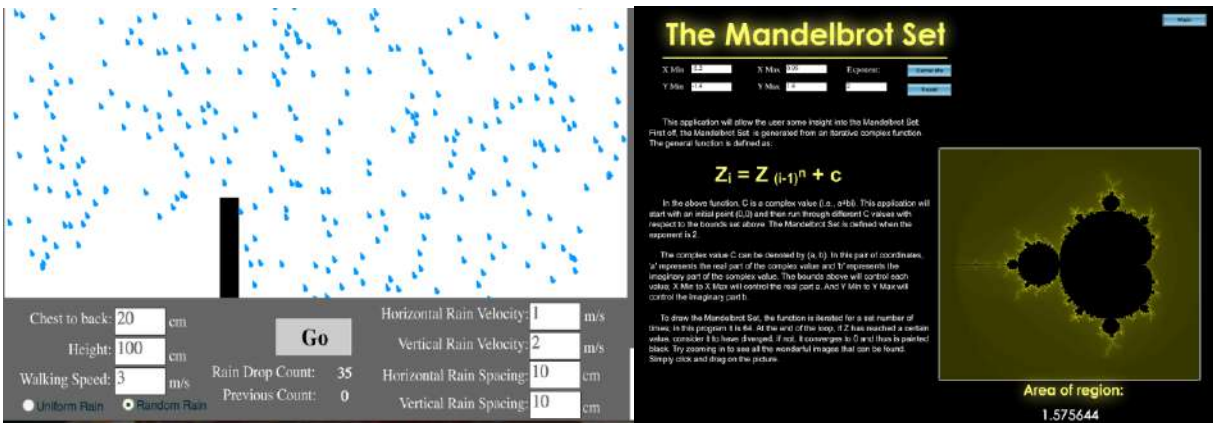

Figure 1. To the left, Matthew and Kylie's real-world situation microworld: "is it better to walk or run in the rain?"; to the right, Adam's pure mathematics microworld about the bounded area of the iterative complex function defining the Mandelbrot set as the exponent increases.

This individual project-based activity requires a completely different teaching format. Because students select the topic of their individual microworld projects, this is a demanding, 'risky' approach for the instructor, in particular in regards to the instructor's curriculum script, resource system, and time economy features (Ruthven, 2009). The change in the instructor's activity format and working environment is not challenging: the instructor acts 'on-demand' for students, and this individual guidance takes place in labs or at unscheduled time.

During the original project period, details of the instructor's curriculum script could not have been elaborated ahead of time as it is driven by the students' selection of topics. The instructor may need to engage in research for some student projects: mostly to evaluate the feasibility of the project, which possibly involves a simplification of a proposed topic or problem and to guide students to adequately research the topic. The instructor may also be called upon to guide and to assist in the designing and running of the investigation. This is demanding as it is different and possibly new for each project and for each time the course is offered, and is part of the "at risk" aspect for the instructor. In terms of resource system, the instructor mostly relies on his/her research skills. This involves for each student project to help identify 'on the spot' good resources at the right level for students. As for instructor's time economy, this part of the MICA courses is very much time demanding because of the individual guidance and research component required by the instructor for each student project (one of the reasons why MICA course sections are capped to 35 students). The evaluation of these projects is no less demanding.

Concluding Remarks

Throughout this practice report paper, we described how the teaching at Brock University of the MICA courses embraces a constructionist paradigm based on using and developing computational thinking for mathematics. By course design, MICA students learn by making through individual projects in the form of designing, programming, and using a mathematics microworld (i.e., a tangible computer object that is shareable) to learn and do mathematics. This is what we have called the 'microworld approach' in this paper. The student-centred constructionist learning experience in MICA courses culminate to individual original projects for which students select their own topics, thereby have the opportunity of it being meaningful to them. As such, the design aims to empower students to work as mathematicians. In terms of mathematics content, the microworld approach outlining the MICA courses has impacted the 'what' of mathematics (Buteau et al., 2016), a key characteristic distinguishing constructivism from constructionism (Noss & Clayson, 2015).

In order to reflect the design of the course, instructors have to change their pedagogy in a significant way, which highlights many characteristics of constructionist teaching. For example, MICA instructors tend to act more as facilitators than lecturers, and to use a slower pace to cover mathematics content. When appropriate, they make transparent the messy development of mathematical ideas. This is a shift from a traditional instructionist approach towards one of empowering students, thereby aligning with the aims of the courses (Buteau et al., 2015). In Buteau et al. (2016), we examined the learning experience of a student, named Ramona, through her three MICA courses. We concluded that "students such as Ramona engage in constructionist experiences of mathematics learning…," adding that "[s]tudents also progressively develop proficiency in the third pillar of scientific inquiry mentioned by the European Mathematical Society (2011)," namely: "Together with theory and experimentation, a third pillar of scientific inquiry of complex systems has emerged in the form of a combination of modeling, simulation, optimization and visualization" (European Mathematical Society, 2011, p.2).

The present paper discussed the roles of and demands on the university instructors teaching the MICA courses. The application of Ruthven's (2009) model on the professional adaptation of school classroom practice with technology proved to be very insightful as to identify key components of instructors' roles and demands specific to the constructionist approach in the MICA courses. However, we suggest that, at the university level, a 'Teaching Assistant' structuring feature be added to Ruthven's (2009) model.

There seem to be relatively few sustained implementations of microworlds in mathematics instruction (Healy & Kynigos, 2010). In this paper, we identified in the mathematics MICA courses, implemented at Brock University since 2001, that the 'meaningful to students' constructionist principle impacted, in a significant and challenging way, three of the five key structuring features of the instructor's classroom practice as given in Ruthven's (2009) model, namely curriculum script, resource system, and time economy. This could highlight why the implementation of a constructionist approach by (university) instructors is so challenging.

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