CTmathU
Computational Thinking in University Mathematics
ctuniversitymath.ca

Learning to use programming as a tool for pure and applied mathematical inquiry: What university math students value most from their experience

Jessica Sardella (BSc/BEd Mathematics)

Brock University

Introduction:

- There has been a growing interest in studying and implementing innovative approaches in undergraduate mathematics, such as inquiry-based mathematics education (Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019). Programming is one way to support certain inquiry-based approaches, where students engage in computational practices used by some professional mathematicians (Weintrop et al., 2016).
- Some undergraduate mathematics curricula have integrated programming in this sense. e.g. Brock University has implemented a course sequence called Mathematics Integrated with Computers and Applications (MICA).
- In their recent "Call for Research that Explores Relationships between Computing and Mathematical Thinking and Activity," Lockwood and Mørken (2021) argue that "serious consideration of machine-based computing [including programming] is largely absent from much of our research in undergraduate mathematics education" (p. 2). They suggest that much more needs to be investigated, including about student learning and thinking, and teaching. They also point out that the various approaches to integrating computing in university mathematics classrooms should provide opportunities for research.

(Taken from Broley, 2022)

The MICA Courses:

- The sequence of three MICA courses (I-II-III/III*), implemented at Brock University since 2001, consists of 14 projects (including final projects on student-selected topics) in which students design, program, and use interactive computer environments to investigate mathematics concepts, conjectures, theorems, or real-world situations (Buteau et al., 2015).
- *MICA III* is specifically designed for future teachers.*

Research by the Team:

- In an extended 5-year S.S.H.R.C.-funded (2017-22) research study, the team, led by Prof. C. Buteau (Brock Department of Mathematics and Statistics), has addressed the gap mentioned by Lockwood and Mørken (2021) using the opportunity provided by the 'natural' MICA environment (i.e. data was collected in MICA courses without teaching interventions from the researchers).
- The team examines the teaching and learning of using programming for engaging in pure or applied mathematical inquiry and aims to better understand how university students learn to use programming as a meaningful tool for mathematics ('as mathematicians do'), and how instructors and universities can support that learning (Buteau et al, 2018).
- Using the Instrumental Approach (Rabardel, 1995/2002) as the main theoretical framework, the team has described how university students develop, through a process called 'instrumental genesis', so-called 'schemes' (i.e., actions for certain goals, and beliefs and understanding that drive those actions; Vergnaud, 2009) to effectively use programming (artefact) to conduct mathematical investigations; for example, the scheme of *articulating a mathematical process in a programming language* (e.g., Buteau et al., 2019). The team study has included e.g. the identification of the aspects of engaging in programming-based math activities that students find most challenging (such as the scheme aforementioned) and why, and how they handle these challenges (Broley et al, 2022). In addition, the team examined actions and decision making by instructors that aim at steering their students' instrumental geneses of using programming for mathematical investigations (Buteau et al., 2023), i.e., what is called the 'instrumental orchestration' (Trouche, 2004). For example, the team identified orchestration features that students found effective in supporting their instrumental genesis, such as different types of individual interventions in labs and the need for a non-judgemental learning environment (Broley et al., 2022).

My Study

- This exploratory study aims to expand the research on student learning, particularly about what students may value the most from their learning experience through engaging in programming-based mathematical inquiry projects.

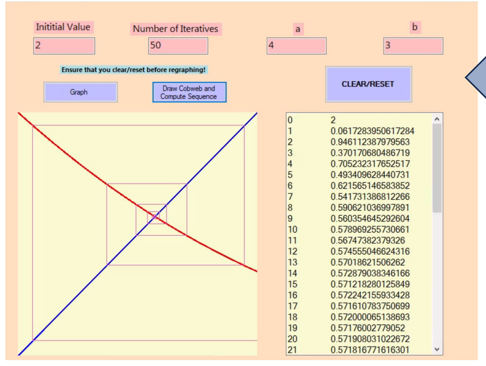
Preliminary Research Question*:

What do undergraduate math students perceive they learn from their engagement in programming-based mathematical inquiry projects?
**see "What's Next" section below for more*

Methodology:

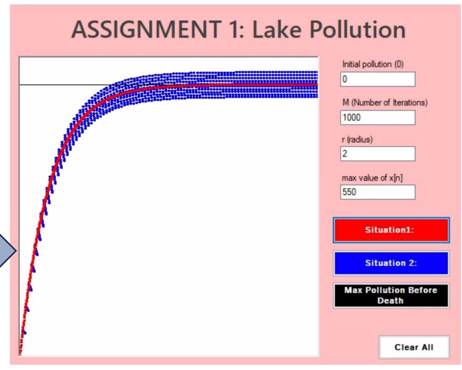
- Participants (n=30): MICA II- n=8, MICA III- n=3, MICA III*- n= 19
- Data in this study includes participants' responses to a post-course online questionnaire for MICA II, III, and III*, specifically short responses to the question asking:
  "Over all the [MICA I, II, III, III] courses, what is your biggest take-away? Please explain:"*
- The qualitative data analysis involved:
  - responses being coded individually by two coders and then consolidated;
  - codes being sorted into themes and consolidated; and
  - themes being sorted into larger categories (Creswell, 2014).

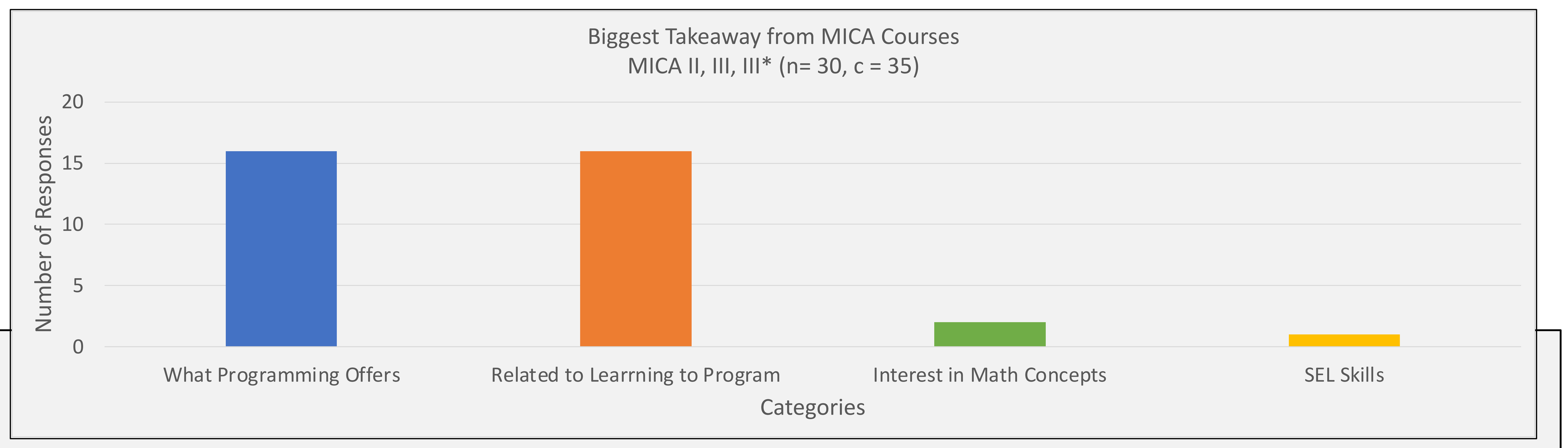
**Responses may fall into multiple themes/categories.*
Some themes/categories emerged from only one response. We want our results to represent each participants' voice and so we still consider this valuable, especially since we aim to identify key aspects of learning outcomes* ***perceived by students.



Screenshot of a MICA I student's program focused on exploring dynamical systems through graphing features including a cobweb. Users input a cubic equation and explore its dynamics over a given interval. They use the cobweb to find fixed points and their nature, as well as the convergence or divergence of the system.

Screenshot of a MICA II student's program relating to discrete equations and involving the modelling of pollution present in lakes. Students were given different situations regarding the flow of water in and out of the lake and created a program to decide which is best for the fish population.





Results:

- 4 categories and 14 themes emerged, summarizing the main perceived key aspects of learning outcomes portrayed by students' 'biggest takeaways' from their experience in the MICA courses.

What Programming Offers

Learning that/how programming is applicable/useful to math

- Participants acknowledge that programming has applications/importance in mathematics and/or that programming can be used within mathematics. Some responses elaborate further by indicating specific ways that programming proves to be useful for mathematics.
- *"How programming can be used to solve math problems" (III)*

Subtheme: Programming can help gain a deeper understanding of mathematics

- ***"Coding is a great way to gain a deeper understanding of math*** *and is a great application tool to learn" (II)*

Understanding that programming is useful (including for math)

- Participants refer to programming (or computers) as a useful/beneficial tool both in the real-world and for math.
- *"My biggest take away is that computers can do a lot. I have learned that computers are a time saver and can be very beneficial in math." (II)*

Understanding there are different ways to do math

- Participants mention that programming allows for a different perspective of math, that math can be presented in many ways (not just traditional methods), or that there may be multiple ways to approach a problem.
- *"Programming is applicable to solving practical problems in mathematics, and* ***can be useful for studying analytically hard mathematical problems from a heuristic perspective.****" (II)*
- *"My biggest take away is that there is no right answer. A problem can be solved in a variety of different ways and you need to be open to that." (III*)*

Understanding programming for math learning (teaching math)

- Participants mention that programming offers a new way to teach mathematics (as opposed to more traditional methods), that they can see how programming can be used in math classrooms, and/or offer their perspectives on how to do so (e.g. slowly, not all at once).
- *"The importance of programming in math. I found this very important as it helped you gain a different perspective of the actual math that you are that you already know.* ***This also helped you gain a different perspective on teaching the actual math so this gave a more hands-on approach to math that is usually done with a white board and marker****." (III*)*

Knowing that/how programming is applicable to the real-world*

- Participants mention more generally the use or applicability of programming in the real-world.
- *"Programming is growing and developing to be a useful tool many should have experience with and can be very beneficial and efficient to use in the real-world." (III*)*

Interest in Math Concepts

Future further research in math concept (outside of course)

- *"I'd like to learn more about post-graduate research in the field of nonlinear dynamics." (II)*

Past Further research of math concepts (within course)

- *"My biggest take-away from these courses was my chance to learn about interesting math concepts…I found every math topic that was gone over to be at least a little bit interesting and many times even looked into them further on my own time…" (III)*

Related to Learning to Program

Learning/gaining programming skills

- Participants indicate gaining the knowledge of how to program (or improving their programming skills), understanding of code, and/or increased comfort in programming.
- *"Overall being comfortable with coding and building my skills in a supportive environment" (III)*

Awareness of the process of learning to program

- Participants acknowledge that programming is difficult, takes time, requires help, and/or requires perseverance, offer suggestions/tips for learning to program (e.g. work with peers, work one step at a time, small details are important, needs to be introduced slowly), and/or offer encouragement (e.g. it is worth it, eventually it will work out, eventually you will figure it out, etc.)
- *"Keep working at it and eventually it will work out, the small details are just as important than the big picture." (II)*

Perception of programming

- *"My biggest take-away is that coding is not that bad, even if I still don't particularly enjoy it." (III*)*

Enjoying programming

- Participants mention that they enjoyed programming.
- *"I enjoyed practicing my programming skills" (II)*

Developing computational thinking skills

- Participants indicate gaining computational thinking skills.
- *"It's the* ***computation[al] thinking part. It helps me think algorithmically*** *and to use that also in other life situations." (III*)*

Knowing the importance of computational thinking

- Participants indicate learning about the importance of computational thinking.
- *"…I think that these courses helped me improve my ability to problem solve and break apart a question to understand how to properly start outlining a solution for them." (III)*

Knowing that/how programming is applicable to the real-world*

- Participants mention more generally the use or applicability of computational thinking in the real-world.
- ***"It's the computation[al] thinking part. It helps me think algorithmically*** *and to use that also in other life situations." (III*)*

SEL Skills: Can Always Learn More about Self as Learner

Can always learn more about self as learner

- *"That no matter your stage in your education, you can still learn more about yourself as a learner." (III*)*

Preliminary Discussion:

1. The most common "biggest takeaway" reported by students was 'learning that/how programming is applicable/useful to mathematics', followed by 'programming is useful.' This suggests that the greatest key learning outcome perceived by students from their engagement in programming-based mathematical inquiry projects may be the usefulness/applicability of programming for math purposes and/or more generally.
   - In other words, this could be interpreted as some students' perceived appropriation, to a certain extent (i.e. at different stages of their instrumental genesis), of programming as a meaningful instrument for mathematics (Buteau et al., 2019)
2. It seems that most responses related to programming in some way (blue and orange), perhaps meaning that participants found the programming aspect of the course to have a larger lasting impression than the actual math concepts. As mentioned by the participants, programming offers a new way to do math. Since other math courses do not incorporate programming in this way, most students likely haven't experienced it before, and so it would have stood out to them more than the specific math content learned (which students are more accustomed to learning).
3. While the large presence of the 'What programming offers' category is not surprising as this directly relates to students' instrumental geneses, the 'Related to learning to program' category highlights that it is not just the outcome that is perceived by students as important, but also the process.
   Some students also mention learning and taking interest in the mathematics explored in the courses, even at times researching on their own. This connects to recent preliminary results by the team highlighting that for students, learning through engaging in a programming-based math inquiry involves both predicative ('facts') and operational ('put into action') knowledge.
   The presence of SEL skills theme in students' perceptions of their '*biggest* takeaway' is surprising, also because it is not conceptualized per se in the team's current main conceptual framework, but aligns with their recent development to expand it through a combination with Lave and Wenger (1998)'s theory of *Community of Practice* in which the concept of (math) identity development (including SEL skills) also frames 'learning'. (Gueudet et al, in press)

What's Next:

- Going forward, the team may wish to further explore the idea of empowerment that programming may provide to undergraduate mathematic students.
- **Taking Papert (1980)'s concept of "using programming as an 'Object-to-Think-With'", the results of this study could suggest that students may feel a sense of empowerment as a result of use of programming as a tool for their mathematics learning.**
  - **"A powerful way to learn coding/computer studies is to … use the computer as an 'object-to-think-with' (Papert, 1980), … Computing is a source of power to do something and figure things out, in a dance between the computer and our thoughts." (Barba, 2016)**
- This would further respond to the 'Call for Research' by Lockwood and Mørken in which they "wonder if, for some students, opportunities to engage with numerical and algorithmic methods might expand their view of mathematics and empower them in their mathematical thinking and activity." (p.8)

Limitations:

- Small sample size (n=30, c=49) and voluntary participation (results are not necessarily representative of all MICA students)
- Uneven sample sizes between courses

References:

Artigue, M., & Blomøj., M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education, 45*, 797-810
Barba, L. A. (2016, March 5). Computational thinking: I do not think it means what you think it means. Lorena A. Barba Group. https://lorenabarba.com/blog/computational-thinking-i-do-notthink-it-means-what-you-think-it-means/
Broley, L., Buteau, C., Levay, D., Marshall, N., Muller, E., & Sardella, J. (2022). Students facing and handling challenges in programming-based mathematics inquiry projects. *Proceedings of the 2022 Annual Conference on Research in Undergraduate Mathematics Education* (pp. 63-71). Boston, MA.
Broley, L., Ablorh, E., Buteau, C., Mgombelo, J., & Muller, E. (2022). Effective Orchestration features of a project-based approach to learning programming for mathematics investigation. *Proceedings of INDRUM 2022 Fourth Conference of the International Network for Didactic Research in University Mathematics* (pp. 592-601). Hannover, Germany. October 2022.
Buteau, C., Gueudet, G., Muller, E., Mgombelo, J., & Sacristán, A. (2019). University Students Turning Computer Programming into an Instrument for 'Authentic' Mathematical Work. *International Journal of Mathematical Education in Science and Technology,* 51(7), 1020-1041. DOI: 10.1080/0020739X.2019.1648892
Buteau, C, Muller, E., Mgombelo, J., & Sacristán, A. (2018). Computational Thinking in University Mathematics Education: A Theoretical Framework. *Proceedings of Research in Undergraduate Mathematics Education conference* (pp. 1171-1179), San Diego (USA), February 2018.
Buteau, C., Muller, E., & Ralph, B (2015). Integration of Programming in the Undergraduate Mathematics Program at Brock University. In the Online Proceedings of *Math+Coding Symposium*, London (Ontario), June 2015.
Buteau, C., Muller, E., Santacruz Rodriguez, M., Mgombelo, J., Sacristán, A., & Gueudet, G. (2023). Instrumental orchestration of using programming for authentic mathematics investigation projects. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era* (2nd Edition) (pp. 289-322). Springer Netherlands.
Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Sage.
Gueudet, G., Buteau, C., Broley, L., Mgombelo, J., Muller, E., Sacristán, A. I., & Santacruz Rodriguez, M. (in press). Learning programming for mathematical investigations: an instrumental and community of practice approach. *Research in Mathematics Education Journal.*
Laursen, S.L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education, 5*, 129-146
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York, NY: Cambridge University Press.
Lockwood, E., & Mørken, K. (2021). A call for research that explores relationships between computing and mathematical thinking and activity in RUME. *International Journal of Research in Undergraduate Mathematics Education,* 1-13.
Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas.* New York, NY: Basic Books.
Rabardel, P. (1995/2002). *Les hommes et les technologies; approche cognitives des instruments contemporains.* Paris, France: Armand Colin.
Trouche, L. (2004). Managing complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281-307. doi:10.1007/s10758- 004-3468-5
Vergnaud, G. (2009). The theory of conceptual fields. *Human Development, 52*(2), 83-94. https://doi.org/10.1159/000202727
Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouill, L., & Wilensky., U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and technology, 25*, 127-147

SSHRC CRSH
Social Sciences and Humanities Research Council of Canada
Conseil de recherches en sciences humaines du Canada